Price-related Sensitivities of Greenhouse Gas Intensity Targets

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Abstract: Greenhouse gas intensities are an appealing tool to foster abatement without imposing constraints on economic growth. This paper shows, however, that the computation of intensities is subject to some significant statistical and conceptual problems which relate to the inflation proofing of GDP growth. It is shown that the choice of price index, the updating of quantity weights and the choice of base year prices can have a significant impact upon the commitment of intensity targets.

Keywords: Carbon intensity, CPI, inflation proofing, index number problem.

JEL Classification: Q43, C43.

1. Introduction[3]

Climate policy in terms of emission intensity targets is typically expressed as percentage reductions from some base year level. The current United States climate change strategy, for example, aims to reduce the greenhouse gas intensity of the U.S. economy by 18 percent in the next ten years. The key parameter of this sort of mitigation policy is the growth rate of the intensities in question. As it happens, this growth rate is sensitive to a number of national accountancy choices.

Best and widest known among them is the sensitivity regarding the choice of GDP measure. As long as there is no need to compare intensities across different nations, relative changes of intensities are naturally measured in local currency GDP figures. International comparison, however, requires a common denominator: market-exchange rate, purchasing power parity or local currency. Between 1987 and '96, the CO_2-intensity of the US economy, for example, declined either significantly (11

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percent) or very little (1 percent), depending on whether it is measured in constant 1995 US$ or in constant 1987 international dollars (PPP). Moreover, there is no apparent pattern for the signs and magnitudes of the differences between these measures. Thus in 1989, the mentioned local currency intensity of the US economy decreased by 3 percent, while its PPP counterpart increased by 2 percent. If the GDP measure is not specified in advance, then parties can still try to achieve compliance by convenient accountancy choices precisely because different GDP measures, and hence intensity measures, generally do not grow at the same rate. Choosing local currencies has the disadvantage that no comparison can be made between countries baselines. As a matter of fact, the present US intensity target is strictly speaking not formulated in relative terms: 'In efficiency terms, the 183 metric tons of emissions per million dollars GDP that we emit today will be lowered to 151 metric tons per million dollars GDP in 2012.'[4] At least this formulation leaves open for other countries between using international dollars or local currency.

This paper, however, is about some less widely known but equally significant sensitivities of emission intensity growth rates to national accountancy choices that relate to the measurement of GDP. Using nominal GDP conflates price changes with legitimate improvements in intensity, hence some type of GDP quantity index is required. This index is generally achieved by either using constant prices, or deflating the current price figures with a price index. One may even decide for combination of both by using a Fisher-index. Although inflation-proofed, the resulting real-term and constant-price growth rates are still subject to price related sensitivities: in case of deflating GDP by a price index, they are generally sensitive to the type of price index applied; in case of real term GDP, they are sensitive to changes in the price base year. Using a dynamic general equilibrium model economy, it is shown that these sensitivities can be considerable in both the real-term and the constant price approach. Thus, in the particular model simulation used, a switch between the Laspeyres, Paasche and Fisher index, altered the real-term intensity growth over the time horizon by 45 percent and the same turns out to be true for price base year changes in constant-price calculations.

The key determinant of these sensitivities are accountancy choices, i.e. the choice of price index in the case of the real-term methodology, and the choice of price base year in the case of the constant-price one. *Ex post,* such choices would be problematic in the context of an intensity target mitigation regime if one rejects the option of achieving compliance by mere accountancy manipulations. *Ex ante,* they amount to a price-related gamble which may facilitate compliance, but which may also cancel out to a significant degree real mitigation efforts required for compliance. Under a 'return

[4] see http://www.whitehouse.gov/news/releases/2002/02/climatechange.html

to initial levels' scenario, the choice of price base year in our model economy could make the difference between having to take on an arduous mitigation requirement, and being able to sell a significant amount of hot air, i.e. surplus permits.

The aim of this paper is to create awareness among policy makers and other stakeholders that adopting monetized intensity targets is tantamount to a 'price gamble' – even if the targets are framed in inflation proofed terms – with odds co-determined by national accountancy choices and the price vicissitudes of the market. Winners in this gamble are helped in complying by purely price related effects, while losers' actual mitigation efforts are partially or completely nullified by them.

2. Physical Intensities and the Aggregation Problem

The paradigm among the diverse forms of intensity measures is that of emissions generated in the course of producing a unit of physical output such as a kilowatt hour of electricity, a ton of steel of a certain quality, or a car of a specific type. If $x_n^s(t)$ represents the quantity of a single commodity *n*, produced by a firm (country, sector, production line) *s* at time *t*, and $e_n^s(t)$ is the amount of a pollutant *P* emitted in the course of this production, then the '*physical P*-emission *intensity* of *s* for producing *n* at (during) *t*' is defined as

(2.1) $\Gamma_n^s(t) =_{def} e_n^s(t)/x_n^s(t)$.

This intensive measure allows comparisons of the production of the same firm at different times (t, t'), or of different firms (s, s') producing the same commodity (n) at the same time. The evolution of $\Gamma_n^s(t)$ indicates changes in ecological efficiency of producing *n* by process *s*. Since it relates to a specific commodity and a specific pollutant, such physical intensities may be called *micro-intensities*.

There are two natural extensions of this concept. For one, it can easily be extended to accommodate aggregated time periods. If '$t \oplus t'$' refers to the total duration of the disjoint time periods *t* and t',[5] then the quantity produced in the aggregate period is defined as the sum of the production in the two component periods, and similarly for the pollutants emitted, i.e.

(2.2) $x_n^s(t \oplus t') =_{def} x_n^s(t) + x_n^s(t')$, (2.3) $e_n^s(t \oplus t') =_{def} e_n^s(t) + e_n^s(t')$,

respectively. These 'base aggregations' can be used to define a *joint*- or *aggregate intensity*:

(2.4) $\Gamma_n^s(t \oplus t') =_{def} e_n^s(t \oplus t')/x_n^s(t \oplus t') = [e_n^s(t) + e_n^s(t')]/[x_n^s(t) + x_n^s(t')]$.

This temporal extension of the intensity concept has its obvious analogue with regard to the sector parameter *s* where the aggregate intensity for a 'joint firm' $s \oplus s'$ is defined as:

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[5] '⊕' is here used to represent a 'concatenation' or 'aggregation' of the components on either side.

(2.5) $\Gamma_n^{s\oplus s'}(t) =_{def} e_n^{s\oplus s'}(t) / x_n^{s\oplus s'}(t) = [e_n^s(t) + e_n^{s'}(t)] / [x_n^s(t) + x_n^{s'}(t)]$.

The Aggregation Problem is the problem of overcoming certain constraints imposed on extending these *micro*-intensities to *macro*-intensities which involve aggregates of commodities, say GDP, and aggregates of pollutants, such as greenhouse gases. Simply adding up production and emission outputs is not feasible for this would mean to add up apples and oranges. The base aggregations introduced so-far trivially satisfy these commensurability constrains by involving only one commodity (pollutant), itself taken to be measured by additive (extensive) quantities. But in the case of incommensurables – such as kWh (x_n) and tons of steel ($x_{n'}$), or carbon dioxide (e_n) and methane ($e_{n'}$) – the summation expressions "$x_n^s(t) + x_{n'}^s(t)$" and "$e_n^s(t) + e_{n'}^s(t)$" simply have no meaning.

Traditionally the solution has been to exchange them, as it were, into a common currency. In the case of incommensurable greenhouse gas emissions, for example, this has been achieved through *global warming potentials* which are used to transform quantities of different gases into quantities of 'carbon dioxide equivalent' (CO_2e).[6] Global warming potentials are based on time-invariant scientific principles and – provided that atmospheric physics is not completely rejected – constant over time. This, however, does not apply to the conventional way of turning commodities into commensurables. They are transformed into monetary values by multiplying the physical quantities with appropriate prices $p_n^s(t)$. It is usual to have these prices determined and revealed by market mechanisms, which in the case of firms as productive units means one and the same price for commodities being sold in the same perfect market.

While most, if not all, of the points to be discussed would apply to both types of 'common currency exchanges,' our focus will be on commodity monetization. Accordingly, we shall assume that we are dealing with commensurate emissions. And to simplify the exposition, we shall - without loss of generality - focus the following theoretical deliberations on the 'two-commodity', 'two-period' and single market case (allowing us to dispense with the index '*s*'), i.e. we shall assume that $n = 1, 2$ and $t = t_0, T$, thus leaving us with a set of 12 basic parameters:

$x_1(t_0), x_1(T), x_2(t_0), x_2(T)$	physical quantities ('quantities' for short),
$e_1(t_0), e_1(T), e_2(t_0), e_2(T)$	emissions,
$p_1(t_0), p_1(T), p_2(t_0), p_2(T)$	prices.

For expository purposes, these parameters can be grouped into different types of two-dimensional arrays (vectors), reflecting the situation at a given time *t*:

(2.6) $e(t) = \langle e_1(t), e_2(t) \rangle$, (2.7) $x(t) = \langle x_1(t), x_2(t) \rangle$, (2.8) $p(t) = \langle p_1(t), p_2(t) \rangle$,

[6] For a summary review of the issues surrounding this type of aggregation by way of global warming potentials see, for example, 'Global warming potentials: 100-year time-horizon, 1992-1995' in *Global Change Electronic Edition,* <http://www.globalchange.org/sciall/96jul1d.htm>

referred to as the 'emission-', 'quantity-' and 'price-system' (or '-bundle') in period t, respectively. As temporal sequences, these data vectors form ('emission-', 'quantity-', or 'price-') time-series ('histories', 'projections')

(2.9) $E = \langle e(t_0), e(T) \rangle$, (2.10) $X = \langle x(t_0), x(T) \rangle$, (2.11) $P = \langle p(t_0), p(T) \rangle$,

which, when combined, we shall refer to as 'scenarios': $\Sigma = (E, X, P)$. A scenario can be based either on historical data or on artificial data generated by a dynamic equilibrium model. We have opted for the latter since we did not wish to focus our analysis only on what happened in the past but also on what *could* happen in the future. In economic terms, we consider $\Sigma = (E, X, P)$ as an equilibrium allocation consistent with some basic assumptions which are typical in economic growth modelling.

Without specifying further the characteristics of a particular economy – i.e. technology, preferences and endowments – $\Sigma = (E, X, P)$ has only some very general properties. Actually, any such scenario can be thought of as equilibrium outcome of an appropriately specified model economy.[7] However, since not all specifications of technologies, preferences, etc., are realistic, we confine ourselves to equilibrium allocations generated by a model type which is standard in the integrated assessment and dynamic macroeconomics.[8]

3. A Model Economy with Undesired Emissions

This section describes the theoretical underpinning of the numbers we use to illustrate our argumentation. It provides a laboratory economy that generates pseudo-data which – from a theoretical point of view – mimic a feasible and reasonable economy. The essential economic variables are depicted in Figure 1. The reader not interested in the modelling details can skip this section.

Our laboratory economy is framed in a Ramsey-type growth model. Production comprises n sectors, with each sector producing a single commodity jointly with emissions as an undesired by-product. Technologies are described by production functions

(3.1) $x_n(t) = f_n(k_n(t), l_n(t))$

where $k_n(t)$ and $l_n(t)$ are sectoral capital and labour input. Production f_n exhibits constant returns to scale, positive and decreasing marginal products. The undesired by-production $e_n(t)$ of, say carbon dioxide, is captured by

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[7] See Boldrin and Montruccio (1986) for details.
[8] See Joos *et al.* (1999).

(3.2) $e_n(t) = \psi_n x_n(t)$,

where ψ_n is the emission coefficient. Due to the linearity assumption, ψ_n also denotes the micro-intensity. Production $x_n(t)$ is used either for consumption $c_n(t)$, or as input into capital formation $z_n(t)$,

(3.3) $x_n(t) = c_n(t) + z_n(t)$.

New capital formation $i(t)$ takes place according to a linear technology with constant input coefficients υ_n, total capital is freely allocated between different productions, and capital deteriorates at rate ϕ:

(3.4) $\sum_n k_n(t) = (1-\phi)\sum_n k_n(t-1) + \min\{\upsilon_n z_n^t(t-1), n = 1,...\}$

Savings/Investment decisions are carried out by a representative agent, who maximises the log-discounted sum of Cobb-Douglas type per period utilities,

(3.5) $\sum_t \beta^t \sum_n \alpha_n \ln(c_n(t))$.

By virtue of the welfare theorems,[9] the solution of the maximisation problem can be considered as the outcome of a decentralised market process with a complete set of forward markets. Accordingly, the Lagrange-multipliers associated to the commodity balances (3.3) give a sequence of present value prices $\{ q_n(t), t = 1,... \}$. These prices are a model artefact; they assume a market system where all contracts are concluded at date t=1, even those whose transactions takes place in the far future. Moreover, prices are only determined up to a constant positive multiplier. Hence, one may choose one commodity as numeraire.

Fortunately, present value prices can be readily translated into current prices, which would emerge from a sequence of spot markets. The relative current prices within a period are exactly the relative present value prices. The current prices level, however, is determined by the money supply. This means that we can apply a period specific scaling parameter to translate period t forward prices into period t current prices. The resulting sequence of spot market prices $\{ p_n(t), t = 1,... \}$ depends upon the choice of these scaling parameters. The scaling – in turn – is a monetary policy variable and relates to the per period money supply. We assume that this choice is taken by a monetary institution, which pursues inflation targeting. For technical reasons, we assume that the representative agent holds money balances only for transaction

[9] See Mas-Colell et al. (1995):312-316

purposes and not as device to shift income over time. Hence the model allows for real investments only.

Monetary supply in period t is given with *M(t)*. This translates period t present value prices into current prices according to

$$(3.6) \quad p_n(t) = q_n(t)\frac{M(t)}{\sum_m q_m(t)x_m(t)}$$

Note that (3.6) preserves equal relative prices in terms of current and present value prices as well as absorption of *M(t)*.

To sum up, GDP in *current prices* and total emissions $e(t)$ at t are given by

$$(3.7) \quad GDP(t) = \sum_n p_n(t)x_n(t), \quad (3.8) \quad e(t) = \sum_n e_n(t)$$

This model provides a consistent scenario $\Sigma = (E, X, P)$ for the purpose of illustrating the price-related sensitivities of intensity targets. We calibrated the model parameters such that a structural change patterns occurs. Sector 2 is assumed to experience exogenous technological progress, with the total factor productivity of the Cobb-Douglas specified technology increasing by 15 percent annually over a period of ten years. At the same time, outputs of sector 2 are used more efficiently in capital formation, with the input coefficient υ_2 declining smoothly towards 50 percent of its initial value within ten years. Afterwards, exogenous technical change vanishes and the economy moves towards its steady state. The abolition of technical change from period ten on explains the kinks in Figure 1.

The assumptions with respect to the evolution of parameters induce sector 2 to grow faster than sector 1, and to induce a change of relative prices with prices of commodity 2 declining. We shall make use of a 15-period two-sector simulation run of this model yielding the results shown in Figure 1.

Figure 1 shows that the model economy experiences significant structural change in favour of sector 2. Sector 2 is more carbon intensive than sector 1, hence emissions evolve in between. As growth in sector 2 is accompanied by a decrease in its output price, the value shares in GDP computed in current prices will not necessarily mirror the gaining weight of the second sector.

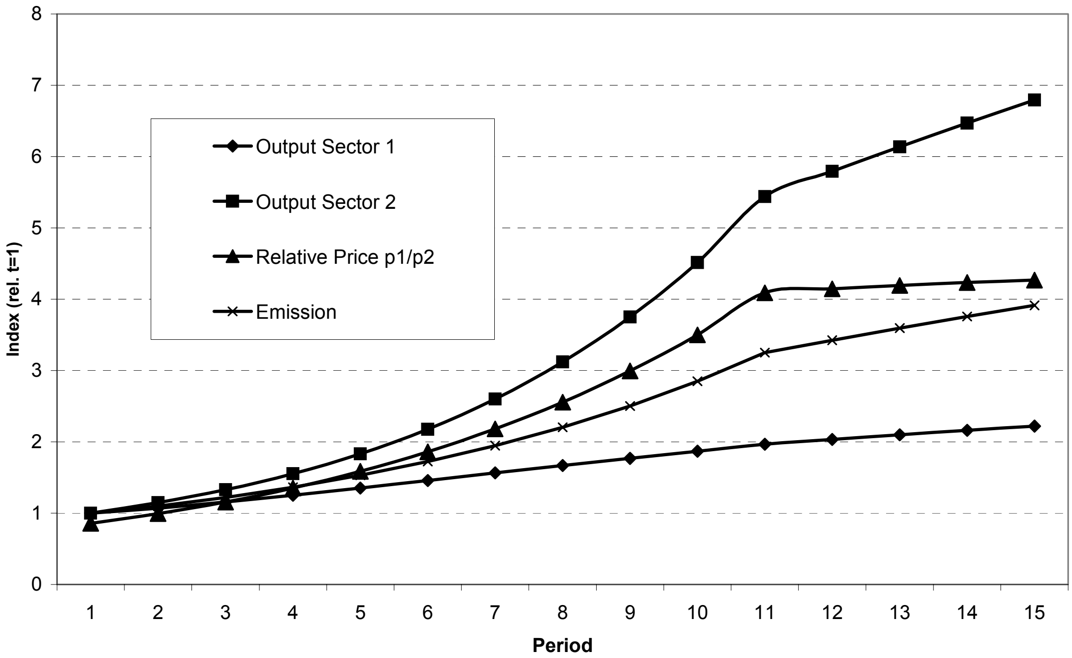

Figure 1: Baseline Simulation Results. Due to technical progress Sector 2 outperforms Sector 1 in real growth rates. Sector 2 is more carbon intensive than sector 1.

4. Current-price Intensities

This section argues that nominal measures of GDP do not work, even if the economy is free of inflation. As suggested by the term, current-price intensities simply employ prices and quantities of the current period. The natural extension of the physical intensity concept is to define the current-price macro-intensity at *t* as total emissions per GDP:

(4.1) $\Gamma(t) =_{def} e(t)/GDP(t)$.

The current price intensity at *t* is thus a function of all the parameters pertaining to this period (and only those), i.e. $\Gamma(t) = \Gamma(\, e(t),\, x(t),\, p(t))$. The *intensity growth rate*

(4.2) $\gamma =_{def} \Gamma(T)/\Gamma(t_0)$

can be described as

(4.3) $\gamma_c = \varepsilon / \pi_c$ – where

(4.4) $\varepsilon =_{def} e(T)/e(t_0)$ and (4.5) $\pi_c =_{def} p(T)\cdot x(T)/p(t_0)\cdot x(t_0)$

are the growth rates of the total emissions and current price GDP, respectively. It is thus a function of all the 12 parameters involved:

(4.6) $\gamma_c = \gamma_c(\, e(t_0),\, e(T),\, x(t_0),\, x(T),\, p(t_0),\, p(T))$.

These current-price intensities are generally eschewed in the context of setting emission mitigation targets for their well-known sensitivity to the inter-temporal price-related phenomenon of inflation: The use of current-price intensity figures is seen as awkward – if not inadmissible – because this price sensitivity allows for the possibility to achieve part of compliance through inflation, which is not deemed to be an acceptable mitigation measure.[10]

To simulate the effects of inflation, we assume the monetary policy in our model to pursue inflation targeting. It aims to stabilize a consumer price index, CPI, at a prescribed level of either zero or two per cent inflation. Applying a Laspeyres based CPI to measure inflation is common practice among most nations (see Lebow and Rudd 2003).

In the first case, the economy is free of inflation in terms of the CPI. The second case is chosen since from a theoretical point of view some inflation seems necessary to allow adjustment of relative prices. It is common practice among most nations to apply the Laspeyres equation in computing the CPI, i.e. to define the CPI as

$$(4.7) \qquad CPI_L(t,t_0) = \frac{p(t)c(t_0)}{p(t_0)c(t_0)},$$

where t_0 denotes the base period, and t the current period.

	GDP Index		GHG Intensity (change from t = 1)		
	Annual inflation rate (CPI)		*Annual inflation rate (CPI)*		*CPI basket up-dated every 5 years*
Period	*0 %*	*2 %*	*0 %*	*2 %*	0 %
5	*1.58*	*1.71*	*0.97*	*0.90*	*0.97*
10	*2.61*	*3.12*	*1.09*	*0.91*	*0.97*
15	*3.32*	*4.37*	*1.18*	*0.89*	*1.00*

Table 1: Current price sensitivity

Table 1 contrasts current-price based sensitivities for two inflation scenarios computed by the model of Section 3. In case one, monetary policy is chosen such that inflation is completely banned. In the second case, prices annually increase by two per cent. The figures demonstrate that a notable but non-alarming inflation rate can change the time path of greenhouse gas intensities dramatically. While in case of absent inflation, intensities increase over time by 18 per cent over 15 periods, an inflation rate of two per cent induces a reduction by 11 per cent. This sensitivity shows that eliminating inflationary distortions of prices is indispensable before they

[10] For examples, see Chapter 3 in Müller et al. (2001).

enter into the calculation of sensitivities. The last column in Table 1 shows the case where the CPI commodity basket is updated every five years and inflation is completely banned again. That is, the CPI of period ten is based upon quantity weights of period five, for example. The figures indicate a huge potential impact of quantity updating.

5. Price-Indices and the 'Index Number Problem'

The CPI as price index is biased as an inflation index since it focuses only on consumption. Alternatively, a price index can be constructed by using GDP weights. However, this approach is subject to economic criticism for there is a fundamental problem in constructing index numbers, known as the *index number problem*. According to *The New Palgrave* "index number" entry,[11] the *index number problem* is to find a pair (*k*) of numbers: a *price* ($P_k(t)$) and a *quantity* ($X_k(t)$) *index* for each period *t* such that their product equals the current-price GDP:

(5.1) $P_k(t) \cdot X_k(t) = p(t) \cdot x(t)$ for $t = t_0, T$.

Given two such pairs of numbers, the current-price GDP growth rate π_c (3.5) can be factorised into a price component: the (*k*-) *inflation* rate δ_k; and a quantity component: the (*k*-) *real-term* GDP growth rate π_k:

(5.2) $\pi_c = \delta_k \cdot \pi_k$, with (5.3) $\delta_k =_{def} P_k(T)/P_k(t_0)$, (5.4) $\pi_k =_{def} X_k(T)/X_k(t_0)$

There are infinitely many combinations of such index pairs and scores of proposals for functional forms to generate them. The most widely used price index formulas, for example, specify the ratio $P_k(T)/P_k(t_0)$. More precisely, they assume a normalised initial price index

(5.5) $P_k(t_0) =_{def} 1$, and then proceed to define $P_k(T)/P_k(t_0) = P_k$:

(5.6) $P_L =_{def} p(T) \cdot x(t_0) / p(t_0) \cdot x(t_0)$ (Laspeyres)

(5.7) $P_P =_{def} p(T) \cdot x(T) / p(t_0) \cdot x(T)$ (Paasche).

In trying to reconcile these two approaches, Arthur Cecil Pigou and Irving Fisher advocated in the early 1920s to take the geometric mean between the two indices, leading to what has become known as the Fisher ideal price index:

(5.8) $P_F = \sqrt{P_L \cdot P_P}$.

The index number problem is the problem of choice between these and all the others.[12] To clarify and facilitate this choice, attempts have been made to axiomatize index number theory. The consistency and independence of the axioms ('tests') listed in Diewert (1998) have been studied in some detail.[13] The index number problem in

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[11] Diewert (1998).

[12] For a presentation of some of the functional forms suggested as price indices see, for example: Fisher (1922)

[13] Eichhorn and Voeller (1976).

this axiomatic approach reduces to the fact that the axioms proposed fail to be categorical, that they admit several quite different solutions. There is hence is no unique 'correct' choice of index numbers. This problem is generic to GDP deflation, and it is inherited by intensities with GDP as denominator.

As already noted, most statistical offices apply the Laspeyres formula in calculating most basic components of their price indices. Among them, the CPI is the major ingredient in measuring the inflation rate. Quantity weights are generally updated in five year intervals. The US Bureau of Labour Statistics dropped this approach in 1999 to substitute it by a superlative index. This change has been advocated by the Boskin Commission with the aim to eliminate the substitution bias. The substitution bias is one of the flaws of all indices based on fixed quantity weights. The basic idea is quite simple. If prices rise, consumers can escape from higher expenditures partially by adapting their commodity basket. They will substitute commodities whose relative prices increased by other products whose relative prices declined. Hence an index like the Laspeyres index will overstate inflation. Actually, it can indicate inflationary pressure though only relative prices have changed. To circumvent the substitution bias, Diewert (1976) showed that there exists a class of superlative indexes which can account for the substitution bias. Among these indices is the Fisher index, whose computation is statistically more demanding than a Laspeyres index.[14] We consider these indices in more detail in the following Section.

6. Real-term Intensities

Given such price indices, our current-price GDP figures can be deflated by different price indices such as the Laspeyres index P_L (5.6), the Paasche index P_P (5.7), or Fisher index (5.8). Given the normalisation (5.5), the quantity index for the initial period is simply the current-price GDP: $X_k(t_0) = p(t_0) \cdot x(t_0)$ for *k* = *L*, *P*; and the deflated – also known as 'real-term' – GDP in the final period is the current-price GDP divided by the final period price index:

(6.1) $X_k(T) = p(T) \cdot x(T) / P_k(T)$.

Thus if $\pi_k =_{def} X_k(T) : X_k(t_0)$ is used to denote the (*k*-) real-term GDP growth rates we find that:

(6.2) $$\pi_L = \frac{p(T) \cdot x(T)}{P_L} : X_L(t_0) = \frac{p(T) \cdot x(T)}{p(T) \cdot x(t_0)}$$ (Laspeyres GDP growth)

(6.3) $$\pi_P = \frac{p(T) \cdot x(T)}{P_P} : X_P(t_0) = \frac{p(t_0) \cdot x(T)}{p(t_0) \cdot x(t_0)}$$ (Paasche GDP growth)

Note that these real GDP growth rates are closely related to the corresponding quantity indices. Quantity indices are computed on fixed base year prices. A

[14] To compute the Laspeyres index, only prices must be tracked over time. The Paasche index requires both, current prices and quantities as input.

Laspeyres quantity GDP growth rate corresponds with the Paasche GDP growth and vice versa. The next section is more concrete on this issue.

The real-term GDP growth in both cases is thus not merely a function of the product bundles $x(t_0), x(T)$, but also of a price vector, namely $p(T)$ in the case of the Laspeyres-growth π_L, and $p(t_0)$ in the case of Paasche-growth π_P. Real-term GDP growth rates, in other words, display what in Section 7 we shall refer to as a 'residual' sensitivity. At this stage, all we wish to emphasise that is that it is not the same as the price-index sensitivity of real-term growth rates which refers to the well-known – but not sufficiently publicised – fact that they generally depend on the chosen price index.

Both the price-index sensitivity and this residual price-sensitivity transfer directly to intensities calculated with real-term GDP figures, leaving us with a Laspeyres and a Paasche real intensity growth:

(6.4) $\gamma_L = \varepsilon / \pi_L = \varepsilon \cdot p(T) \cdot x(t_0) / p(T) \cdot x(T)$ (Laspeyres Intensity Growth),

(6.5) $\gamma_P = \varepsilon / \pi_P = \varepsilon \cdot p(t_0) \cdot x(t_0) / p(t_0) \cdot x(T)$ (Paasche Intensity Growth),

– both sensitive to changes in a relative price system, namely that of the final period in the Laspeyres case: $\gamma_L = \gamma_L(p(T),...)$, and that of the initial period in the Paasche case: $\gamma_P = \gamma_P(p(t_0),...)$. To be noted, in particular, is that the two growth rates will only be identical if there are no changes in the underlying relative prices:

(6.6) $\gamma_L(p(T)) \equiv \gamma_P(p(t_0))$ iff $p(T) = p(t_0)$.

Period	*Laspeyres*	*Paasche*	*Fisher*
5	*- 2%*	*-6%*	*-4%*
10	*+12%*	*-16%*	*-3%*
15	*+22%*	*-20%*	*-1%*

Table 2: Changes in Real-Term Emission Intensities from t = 1

The size of the effect of switching between these indices on the ensuing real-term intensity growth rates (i.e. their *price-index sensitivity*) can be considerable, as our model illustrates. Assuming that the monetary institution supplies money such that the economy is inflation free according to a Laspeyres fixed base year index, we arrive at the following results: While both *L*- and *P-intensities* decrease in the first 5 periods (albeit at appreciably different rates),[15] much more significant differences appear after 10 and 15 periods, when the Paasche emission intensity reaches 20% *below* its initial level, while the Laspeyres intensity has actually changed direction and climbed to 22% *above* the *t* = 1 level. In other words, the price-index sensitivity in this particular example is close to 50 percentage points, a fact which clearly raises a problem in the context of setting intensity based mitigation targets.

[15] Being a mean of these two, the Fisher intensities need not be discussed separately.

7. Constant-price Intensities

Another way of dealing with the sensitivity of current-price figures to any effects of inter-temporal price changes – acceptable or not– is to eliminate their root-cause, as it were, by turning to constant-price aggregations using a single set of prices, say p, in the monetizing aggregation of the quantities in each period. This generates the following growth-rates $\pi(p)$ and $\gamma(p)$ for the aggregate product – the *constant-price GDP* – and for the *constant-price intensities*, respectively:

(7.1) $$\pi(p) = \frac{p \cdot x(T)}{p \cdot x(t_0)};$$ (7.2) $$\gamma(p) = \varepsilon \cdot \frac{p \cdot x(t_0)}{p \cdot x(T)}$$

As concerns these two-period growth rates, the two 'real-term' examples discussed in the preceding section are thus equivalent to constant-price calculations. The Paasche-growth is the same as the constant-price GDP growth in $p(t_0)$-prices: $\pi(p(t_0)) = \pi_P$, and the Laspeyres-growth the same as the constant-price GDP growth in $p(T)$-prices: $\pi(p(T)) = \pi_L$. Unlike in the current-price case, however, constant-price calculations do *not* lead to an 'index number problem'.[16] Their monetized products are what index number theory refers to as *pure quantity indices*.

Yet the fact remains that they too depend on more than the relevant 'quantity scenario' given in the two quantity bundles $x(t_0), x(T)$. And the same remains true of their growth rates: the constant-price aggregate quantity growth rate $\pi(p, x(t_0), x(T))$ is generally *not* fully determined by the data of the underlying quantity scenario, but varies also under changes of the price baseline. Indeed, these aggregate growth rates are price *in*dependent only in a very narrow range of quantity scenarios, namely those in which all sectors happen to grow at the same rate, because it is a mathematical truth that for any $x(t_0), x(T)$:

(7.3) $\pi(p, x(t_0), x(T)) = \pi(p', x(t_0), x(T))$ for all p, p', iff $x_1(T) : x_1(t_0) = x_2(T) : x_2(t_0)$.

Moreover, in those rather specific uniform sectoral growth cases in which the choice of base-line prices does not affect the aggregate quantity growth we also have a certain homogeneity, in the sense that the aggregate will be growing at the same rate as its sectoral components: for $\alpha =_{def} x_1(T) : x_1(t_0) = x_2(T) : x_2(t_0)$ we have $\pi(p, x(t_0), x(T)) = \alpha$. And the corresponding constant-price intensities display the same sensitivities to base-line price variations:

(7.4) $\gamma(p, x(t_0), x(T)) = \gamma(p', x(t_0), x(T))$

for all p, p', iff $x_1(T) : x_1(t_0) = x_2(T) : x_2(t_0)$,

[16] It follows from $p =_{df} p(t_0) = p(T)$, and the 'tests' (axioms) put forward in Diewert (1998: 768) – that the price index has to be constant, i.e. $P_k(t) \equiv 1$ and hence that the quantity index is uniquely given as $X_k(t) = p \cdot x(t)$ (for $t = t_0, T$).

with the exception that they fail to be homogeneous in the above-mentioned sense.[17]

In short, constant-price intensities retain in practically all scenarios a residual price sensitivity which manifests itself in a sensitivity to the choice of price base-line period (i.e. the period determining the price system *p* to be used in the aggregation). As shown in our model scenario (see Table 3), the latter ('price base-year sensitivity') can again be very large. Choosing the price system of the initial period leads to continuous decrease of the (constant-price) intensity of the model economy of 21%, while under the final price system, the economy's intensity steadily increases by 25%. Again, these discrepancies can be highly significant as concerns the effort in trying to achieve a given intensity target.

The differences are smaller if a chain index is applied. Real GDP growth in terms of the Fisher Index is given with

(7.5) $\pi_{FC} = \sqrt{[p(t)x(t)/p(t)x(t-1)][p(t-1)x(t)/p(t-1)x(t-1)]}$

Chain indices are rebased each period to minimise the substitution bias. Table 3 shows the results of chain indexation. The differences are smaller, in particular the order of magnitude of GHG change.

	Laspeyres Index Base Year				**Chain Indices**		
Period	*1*	*5*	*10*	*15*	*Laspeyres*	*Paasche*	*Fisher*
5	*-6%*	*-2%*	*4%*	*5%*	*-4%*	*-3%*	*-4%*
10	*-16%*	*-5%*	*12%*	*17%*	*-3%*	*0%*	*-1%*
15	*-20%*	*-6%*	*16%*	*22%*	*1%*	*5%*	*3%*

Table 3: Base Year Price Sensitivity of GHG Intensities

Every intensity target scenario leads, albeit only *ex post*, to an (implicit) absolute cap on the emissions in question – i.e. an 'Implicit Assigned Amount' (IAA) – given by the product of the target intensity with the target period GDP. For example, in the case of a 'return to initial level' scenario for, say, target period t = 10, we thus have that

(7.6) $a_k(10) = \Gamma_k(1) \cdot y_k(10)$,

with $a_k(10), \Gamma_k(1),$ and $y_k(10)$ referring to the IAA of the target period (t = 10), the initial intensity, and the target period GDP, respectively, all measured in terms of the same GDP type (k). The mitigation efforts required to achieve this target is determined by the relation of this $a_k(10)$ to the reference ('business as usual') emissions in the target period, i.e. to $e(10)$. In the case of our constant-price model economies, for example, we have that the reference emissions in the target period are

[17] Unlike in the case of quantity aggregates, however, price independence is not tantamount with growth homogeneity. In other words, $(e_1(T)/x_1(T)) : (e_1(t_0)/x_1(t_0)) = (e_2(T)/x_2(T)) : (e_2(t_0)/x_2(t_0)) = \alpha$ implies $\gamma = \alpha$ *only* for one particular set of prices, namely $p_n = e_n(t_0)/x_n(t_0)$.

22 percent above the IAA implied by the intensity target in (t=1)-prices ($a_{t=1}(10)/e(10)=1.22$), and 16 percent below the one implied by the intensity target in (t=10)-prices ($a_{t=10}(10)/e(10)=0.84$), and all this for the same real economic variables.

8. Residual Price Sensitivities

The idea of a (residual[18]) price sensitivity in a price dependent formula, say, $\Phi(p,x)$ is simply that its values actually vary depending on the price-argument (in our case a price-system), or, to put it in its negation in conformity with the specific formulation used in the left-had conditionals of (7.3) and (7.4):

(8.1) Φ is price *in*sensitive iff$_{def}$ for all feasible x, p, p'-scenarios $\Phi(p,x)=\Phi(p',x)$.

GDP related intensity targets are price sensitive. Insensitivity would occur only, if different equilibrium price system induce different equilibrium quantities, i.e, if from (p,x) is an equilibrium it follows that (p',x) is not an equilibrium.

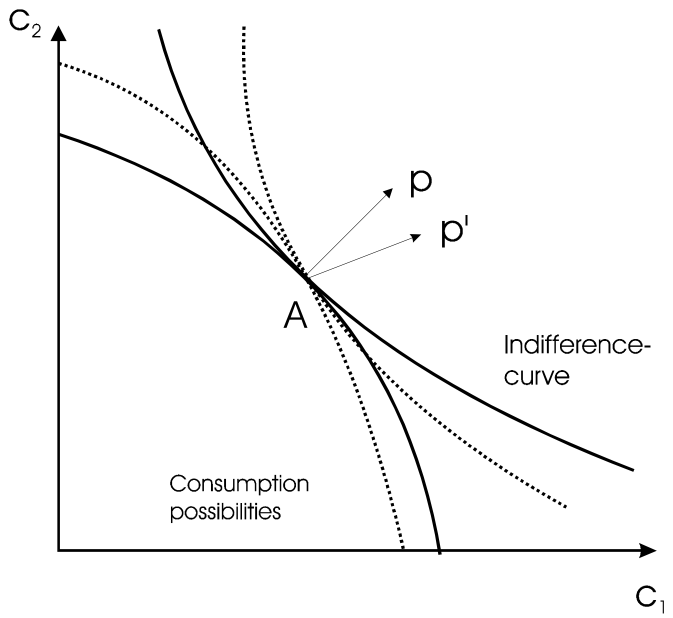

Figure 2: Residual Price Sensitivity

Figure 2 shows an example, where this implication does not hold. A is an equilibrium for both p and p'. This type of non-uniqueness is due to different assumptions with respect to technologies and preferences. We are actually comparing two different economies, yielding the same equilibrium quantities by chance but differing in equilibrium prices.[19] An analysis of the of the effects of this sort of residual price

[18] The epithet 'residual' is simply used to indicate that the sensitivity in question remains even with 'inflation proofed' figures.

[19] Equilibrium prices and quantities are determined by the characteristics of an economy. These characteristics comprise preferences, technologies and endowments. Unfortunately, to date we only

sensitivity of inflation-proofed – real-term, constant price – measures (emission intensities, GDP) on policy making under uncertainty would go beyond the intended scope of this paper and has to be postponed for the time being. At present we only wish to point out that price sensitivities remain an issue, even if one is willing to make the sort of national accountancy choices required to obtain inflation-proofed targets and measures.

9. Synopsis and Conclusions

Having introduced the notion of emission intensity in its most basic, namely *physical* guise, i.e. emissions per physical unit of production, our attention turned to the issue of extending this concept to aggregations. Our focus has been on the method of monetizing incommensurable commodities used to overcome what we termed the 'aggregation problem' to define joint intensities for them. We then proceeded to discuss some of the different versions of such monetizations and their price-related sensitivities, of which we distinguished three tiers.

The first version considered was that of *current-price* monetization, where the physical quantities are turned into monetary values by multiplying them with the prices at the time, i.e. their current price. The resulting current-price intensities are problematic – particularly in setting emission mitigation targets – due to their commonly known sensitivity to inflationary price effects, a sensitivity we classified as belonging to a first tier of the price related sensitivities at issue in this paper.

Two ways in which this first tier sensitivity of current-price figures can be eliminated were considered, namely *real-term* monetization using *price indices* to deflate current-price figures, and monetization in terms of *constant prices*. While 'inflation-proofed', i.e. free of the effects of inflation, the resulting real-term and constant-price growth rates are both subject to a second tier of price related sensitivities, for they are generally sensitive to price index changes (in the case of real-term monetizations) and to changes in the chosen price base year (in the current-price case). Using a dynamic general equilibrium model economy, it was found that these second-tier sensitivities can be very large in both the real-term and the constant price approach. Thus, in the particular model simulation used, a switch between the Laspeyres and the Paasche index, altered the real-term intensity growth by over the time horizon of 45% and the same for changes in price base years in constant-price calculations. Differences in such an order of magnitude are unlikely to be observed in the future, but – at least in principle – they *can* occur in an economy that experiences severe structural change. In

have strong assumptions to ensure the uniqueness of equilibrium prices in our models. Among them is gross substitutability which holds only for very simple preferences structures. For most growth models of the Ramsey type with representative agents, however, uniqueness can be established by the curvature properties of indifference lines. The focus here, however, is thus not on this type of non-uniqueness of equilibria.

our numerical experiments, we assumed that technical progress enhances output growth mainly in one of the sectors with its output price strongly declining as result.

The key determinant of these second tier sensitivities are accountancy choices, i.e. the choice of price index in the case of the real-term methodology, and the choice of price base year in the case of the constant-price one. *Ex post,* such choices would be problematic in the context of an intensity target mitigation regime if one rejects the option of achieving compliance by mere accountancy manipulations. *Ex ante,* they amount to a price-related gamble which may facilitate compliance, but which may also cancel out to a significant degree real mitigation efforts required for compliance. The price sensitivities are partially resolved under chain indices. Then, the *huge* differences in intensities disappear. Nevertheless, there still remains a *significant* difference. Both the first and the second tier of these price-related sensitivities of intensity figures and their growth rates have been known for a long time, although not always equally acknowledged.[20]

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[20] A third tier of price-related sensitivities not analysed in this paper afflicts both types of inflation proofed intensity figures, even once these second-tier accountancy choices are made. Indeed, it is only due to this 'residual price sensitivity' of inflation proofed figures that the second tier sensitivities could arise in the first place. After all, if, say, constant price intensity figures were independent of price system variations, then they would also be insensitive to changes in the price base year.

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11. Appendix: Parameters and Specifications of the computable equilibrium model

Macro-Production

Parameter	Sector 1	Sector 2
Production type	Cobb-Douglas	Cobb-Douglas
Total factor productivity	1 (scaled initial) scenario specific	1 (scaled initial) scenario specific
Capital value share	.3	.5
Sectoral carbon intensity	5.57	2.78

Capital Technology

Parameter	
Production type	Leontief
Input coefficients (initial values)	2 (sector 1) 1.7 (sector 2)
Depreciation rate	.05

Utility

Parameter	
Type	Discounted log-linear over time, Cobb-Douglas in time.
Discount rate	0.05
Expenditure share (sector 1)	.4
Exogenous labour growth rate	.02